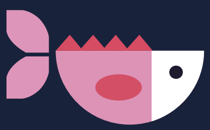

Coalition Control Model: A Dynamic Resource Distribution Method Based on Model Predictive Control

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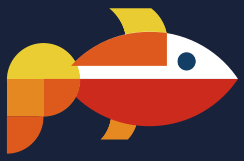
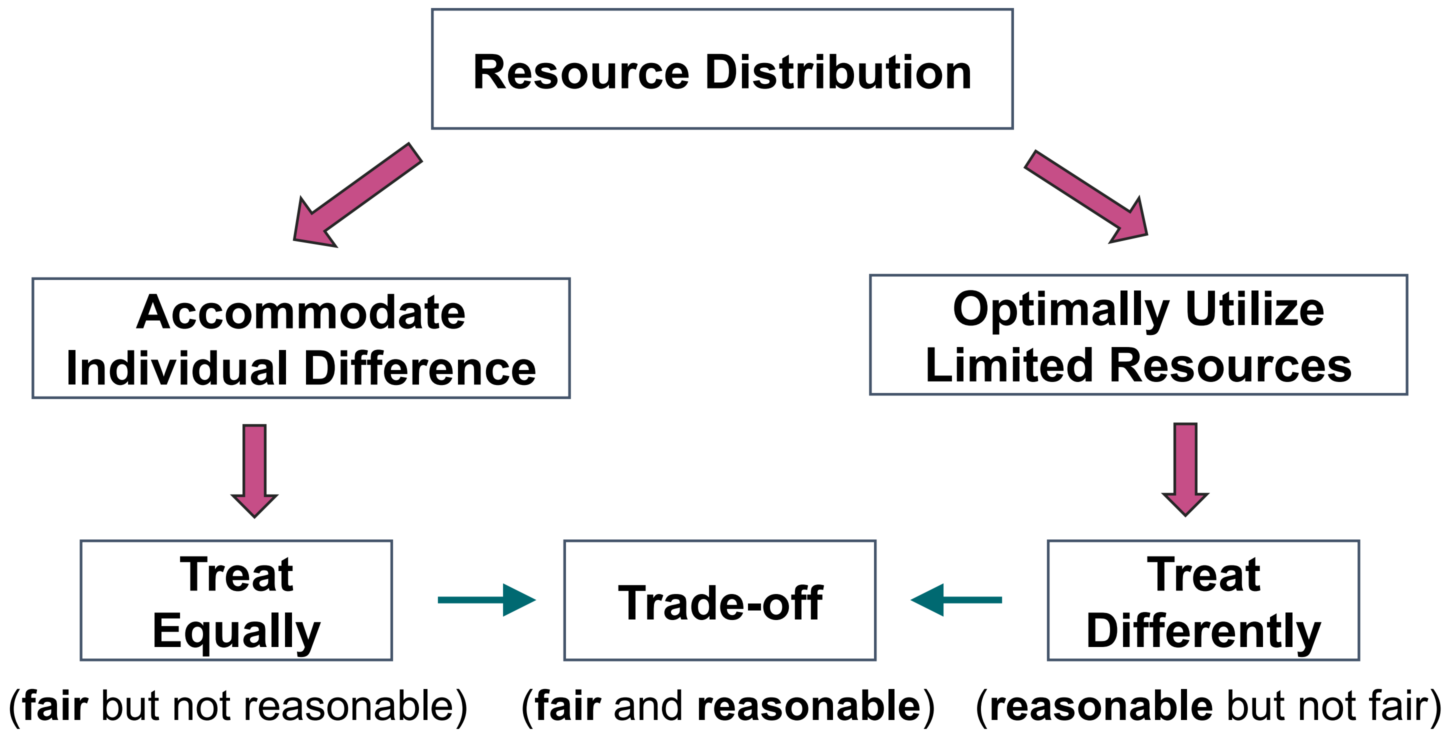
12/12/2020

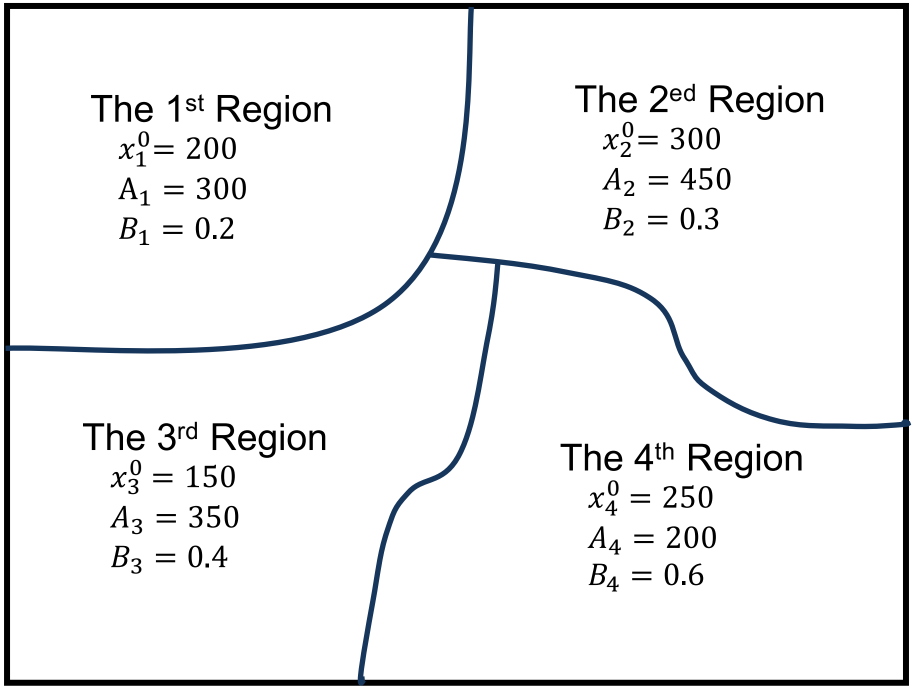


01 INTRODUCTION

The **fair** and **reasonable** distribution of **limited resources** is of great practical importance in solving social problems.



02 MODEL



Fishing Boats

$\gamma_1 = 0.08$

$\gamma_2 = 0.10$

$\gamma_3 = 0.12$

$\gamma_4 = 0.15$

$\gamma_5 = 0.20$

$\gamma_6 = 0.28$

Objective Function

$$O_{c_i} = \sum_t \sum_{k \in C_i} \sum_i \gamma_k e_{k,i}(t)\, x_i(t)$$

$$C_{K(t)} = C_{1(t)} \cup C_{2(t)} \cup \cdots \cup C_{c(t)}$$

$$\begin{aligned} &max \quad O_{C_K} \\ &s.t. \quad \sum_i e_{k,i}(t) = 1 \\ &\qquad e_{k,i}(t) \geq 0, k \in K \\ &\qquad e_{k,i} = e_{j,i}, k, j \in C, C \in C_K \\ &\qquad x_C^{eq} - R < x_C < x_C^{eq} + R \end{aligned}$$

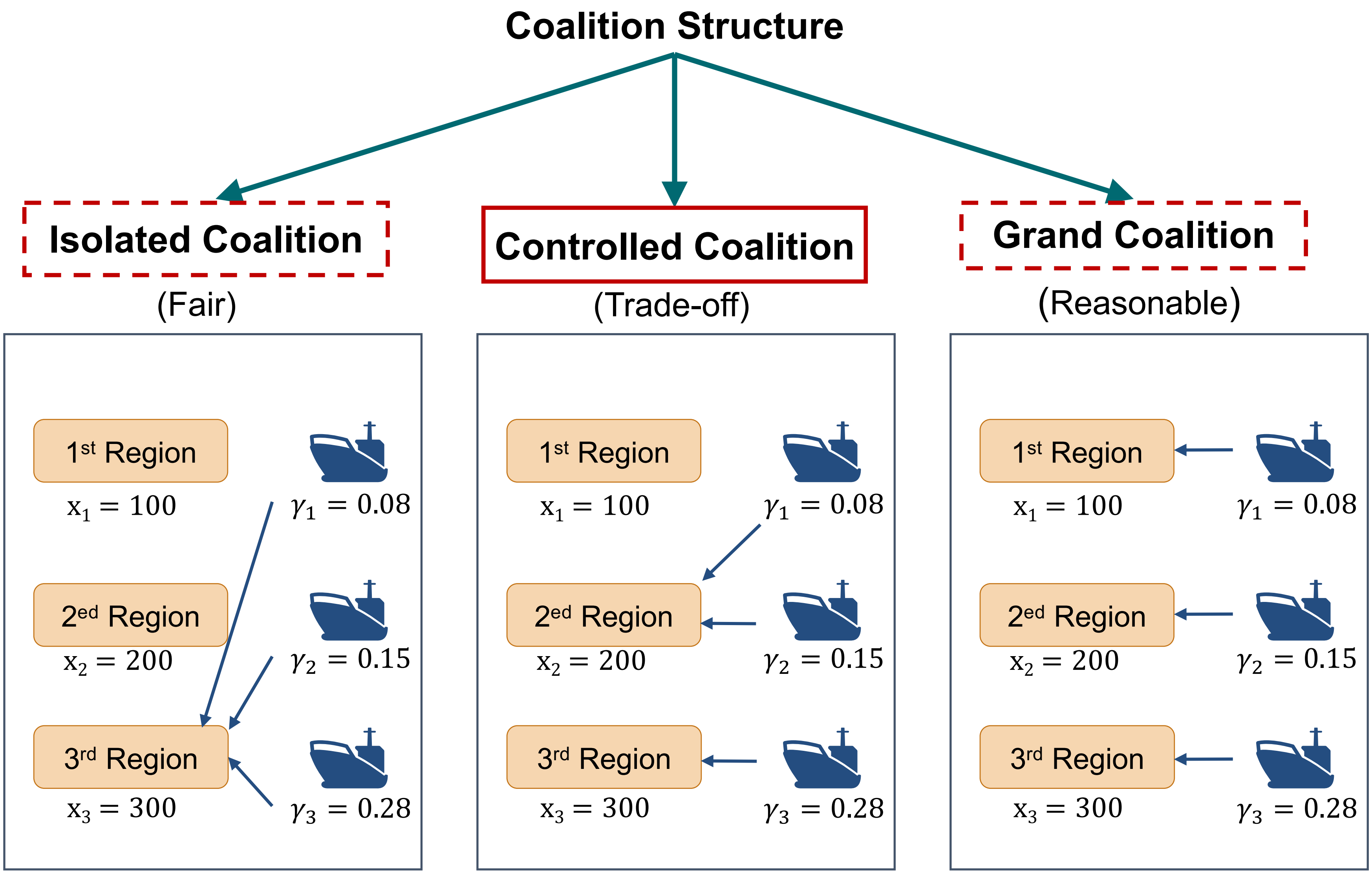
Natural Changes Fish Caught

Evolution Equation: $x_i(t+1) = \boxed{A_i + B_i x_i(t)} - \boxed{\sum_k \gamma_k e_{k,i}(t) x_i(t)}$

A: Fish Inflow
B: Survival Rate
x: Fish Amount
γ: Fishing Capability
e: Fishing Effort
O: Objective Function

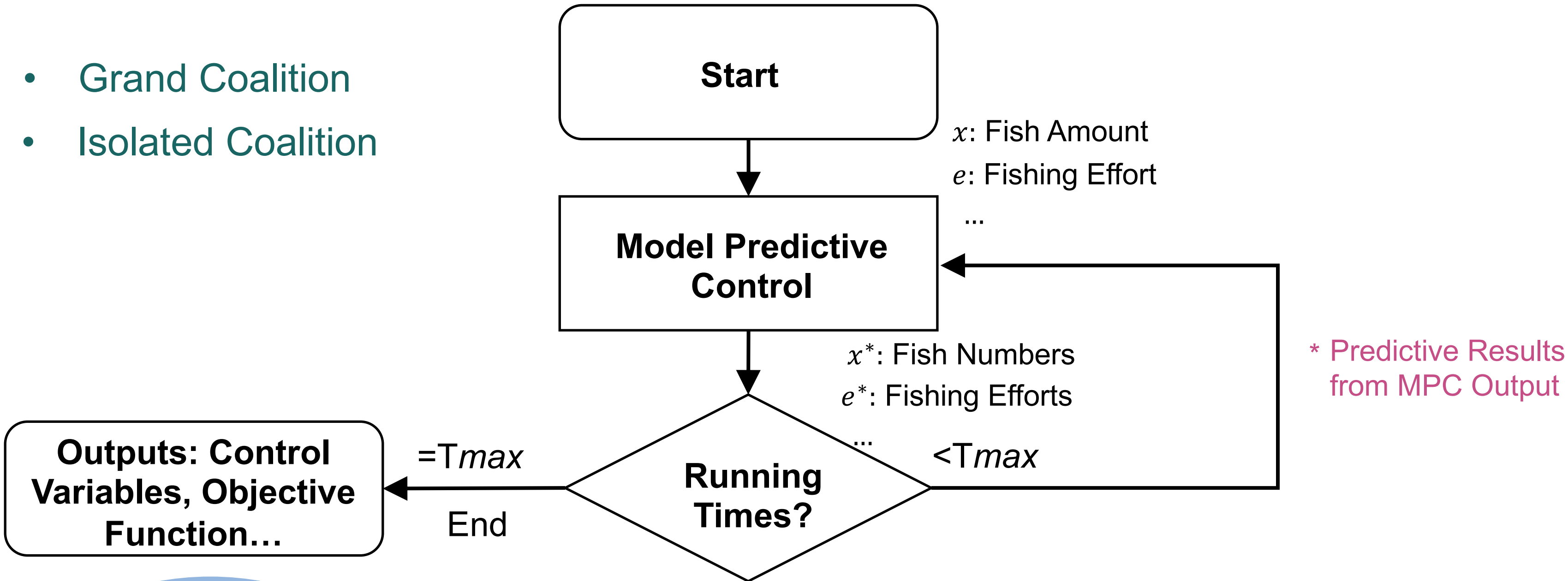
x_C^{eq}:MPC Equilibrium Result
R :Relaxation for Tolerance

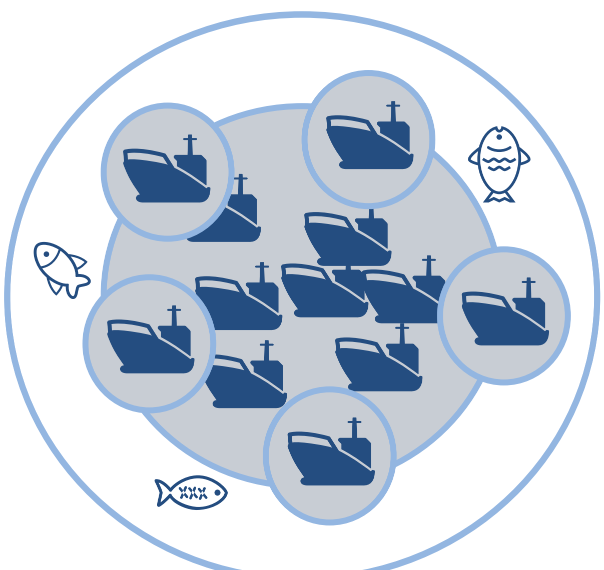
03 METHODS


Coalition Structure
Isolated Coalition
(Fair)
Controlled Coalition
(Trade-off)
Grand Coalition
(Reasonable)
1st Region
$x_1 = 100$
$\gamma_1 = 0.08$
2ed Region
$x_2 = 200$
$\gamma_2 = 0.15$
3rd Region
$x_3 = 300$
$\gamma_3 = 0.28$
1st Region
$x_1 = 100$
$\gamma_1 = 0.08$
2ed Region
$x_2 = 200$
$\gamma_2 = 0.15$
3rd Region
$x_3 = 300$
$\gamma_3 = 0.28$
1st Region
$x_1 = 100$
$\gamma_1 = 0.08$
2ed Region
$x_2 = 200$
$\gamma_2 = 0.15$
3rd Region
$x_3 = 300$
$\gamma_3 = 0.28$

04 ALGORITHM: MULTI-AGENT SYSTEM

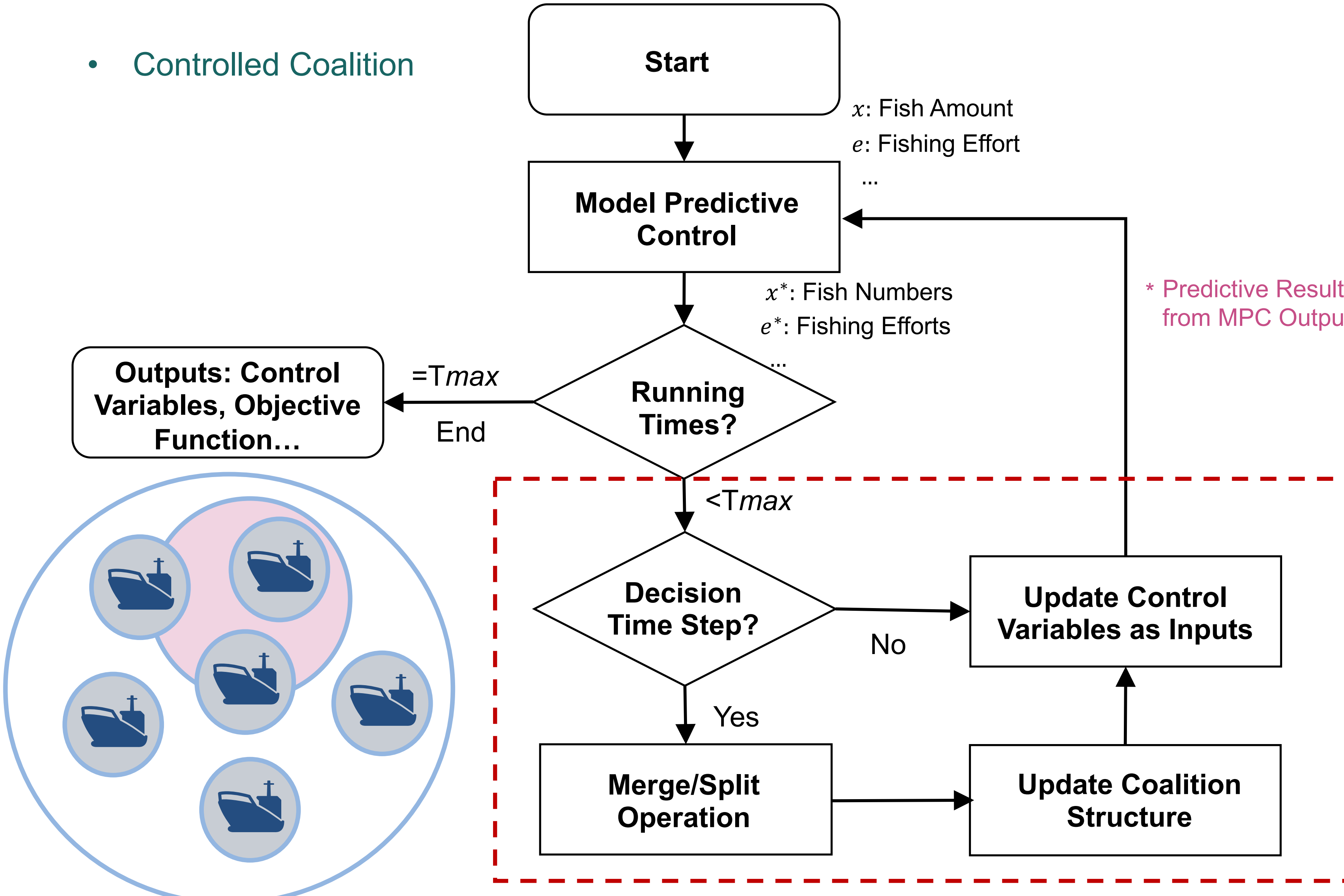
- Grand Coalition
- Isolated Coalition





04 ALGORITHM: MULTI-AGENT SYSTEM

- Controlled Coalition



05 COALITION CONDITION

- Redistribution (A Compensation Mechanism)

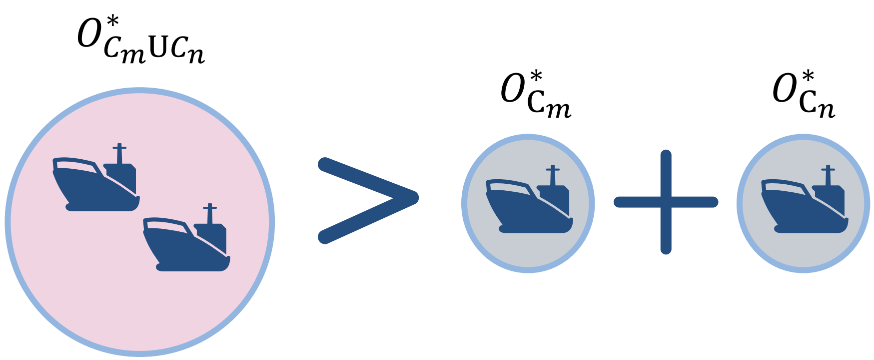
Merge: $O^*_{C_m \cup C_n} > O^*_{C_m} + O^*_{C_n}$

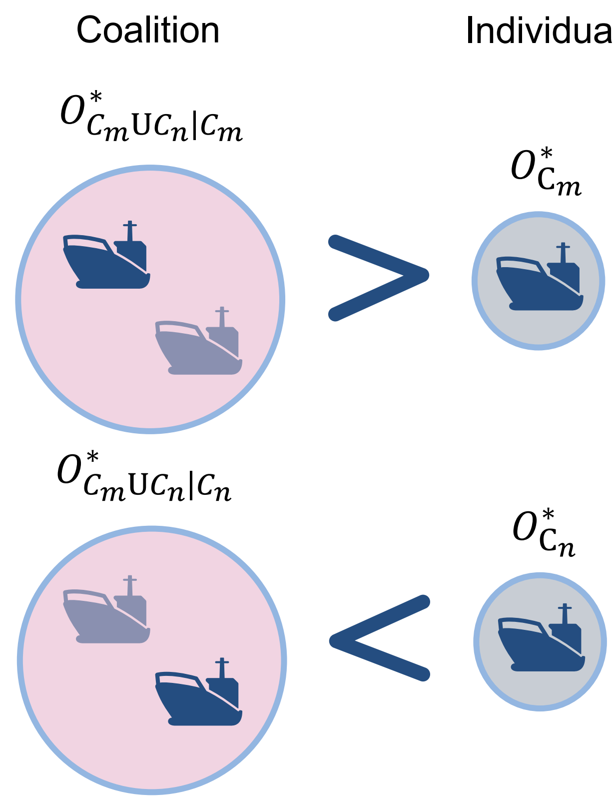
Split: $O^*_{C_m \cup C_n} < O^*_{C_m} + O^*_{C_n}$

Distribute Proportionally:

$$O^*_{C_m \cup C_n | C_m} = O^*_{C_m \cup C_n} \frac{O^*_{C_m}}{O^*_{C_m} + O^*_{C_n}}$$

$$O^*_{C_m \cup C_n | C_n} = O^*_{C_m \cup C_n} \frac{O^*_{C_n}}{O^*_{C_m} + O^*_{C_n}}$$





C_m: Coalition *m* C_n: Coalition *n* O^*: Optimal Fish Caught

$O^*_{C_m \cup C_n | C_m}$: Fish caught of coalition m after m merges with n

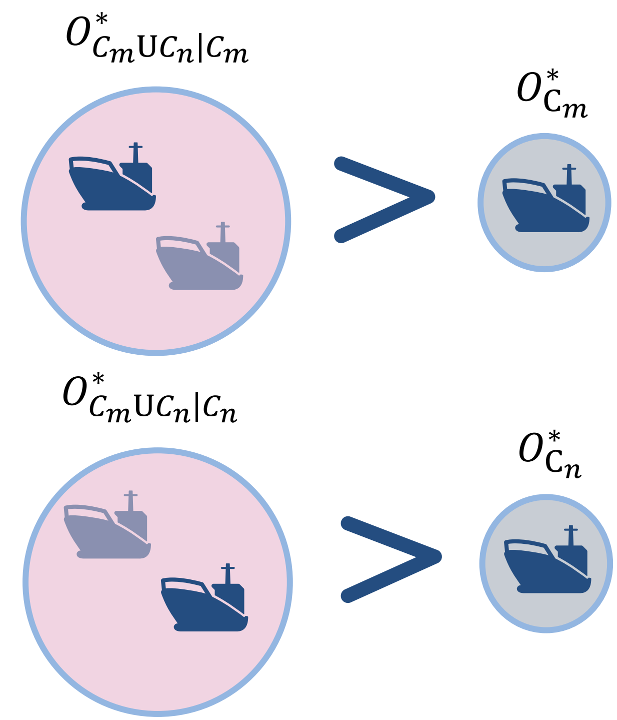
06 COALITION CONDITION

- Without Redistribution

Merge:
$$O^*_{C_m \cup C_n | C_m} \geq O^*_{C_m}$$
$$O^*_{C_m \cup C_n | C_n} \geq O^*_{C_n}$$
AND

$\neq$ $O^*_{C_m \cup C_n} > O^*_{C_m} + O^*_{C_n}$

Split:
$$O^*_{C_m \cup C_n | C_m} < O^*_{C_m}$$
$$O^*_{C_m \cup C_n | C_n} < O^*_{C_n}$$
OR



Fish Amount for Boat m:

$$O^*_{C_m \cup C_n | C_m} = \sum_{k \in C_m} \sum_{i} e^*_{k,i}(t) x^*_i(t)$$

07 HEURISTIC CONTROLLED COALITION

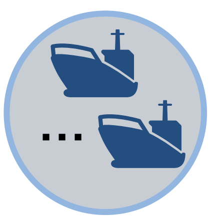
How to determine which boats form coalitions (assuming K boats)

Global Search $\Omega(2^K)$

C_K^2 possibilities

C_K^3 possibilities

...



C_K^{K-1} possibilities

Objective: to find the **largest** post-coalitions fish catch

Local Search $\Omega(K^2)$

C_K^2 possibilities

Objective: to find the two boats with the most **similar** effort vector for a coalition

$$\min_{\mathbf{E},\mathbf{V}} - O_{\mathbf{E}} + \mu \sum_k \|\mathbf{e}_k - \mathbf{v}_k\|_2^2 + \gamma \sum_{1\le k<l\le K} \|\mathbf{v}_k - \mathbf{v}_l\|_2^2$$

$$s.t. \sum_i e_{k,i}(t) = 1, e_{k,i}(t) \ge 0, k \in \mathcal{K}, i \in \mathcal{N}$$

$$x_{\mathcal{C}}^{eq} - \mathcal{R} < x_{\mathcal{C}} < x_{\mathcal{C}}^{eq} + \mathcal{R},$$

[2011 ICML] Clusterpath: An algorithm for clustering using convex fusion penalties

08 RESULT: FISH CAUGHT

Coalition Structure

Isolated Coalition

(Fair)

Description	Fish Value
total fish caught	337.22×10^3
1st boat	29.01×10^3
2nd boat	36.26×10^3
3rd boat	43.51×10^3
4th boat	54.39×10^3
5th boat	72.52×10^3
6th boat	101.53×10^3

Controlled Coalition

(Trade-off)

* With Redistribution

Description	Fish Value
total fish caught	343.00×10^3
1st boat	29.51×10^3
2nd boat	36.88×10^3
3rd boat	44.26×10^3
4th boat	55.32×10^3
5th boat	73.76×10^3
6th boat	103.27×10^3

* Without Redistribution

Description	Fish Value
total fish caught	342.84×10^3
1st boat	29.49×10^3
2nd boat	36.86×10^3
3rd boat	44.24×10^3
4th boat	55.30×10^3
5th boat	73.73×10^3
6th boat	103.22×10^3

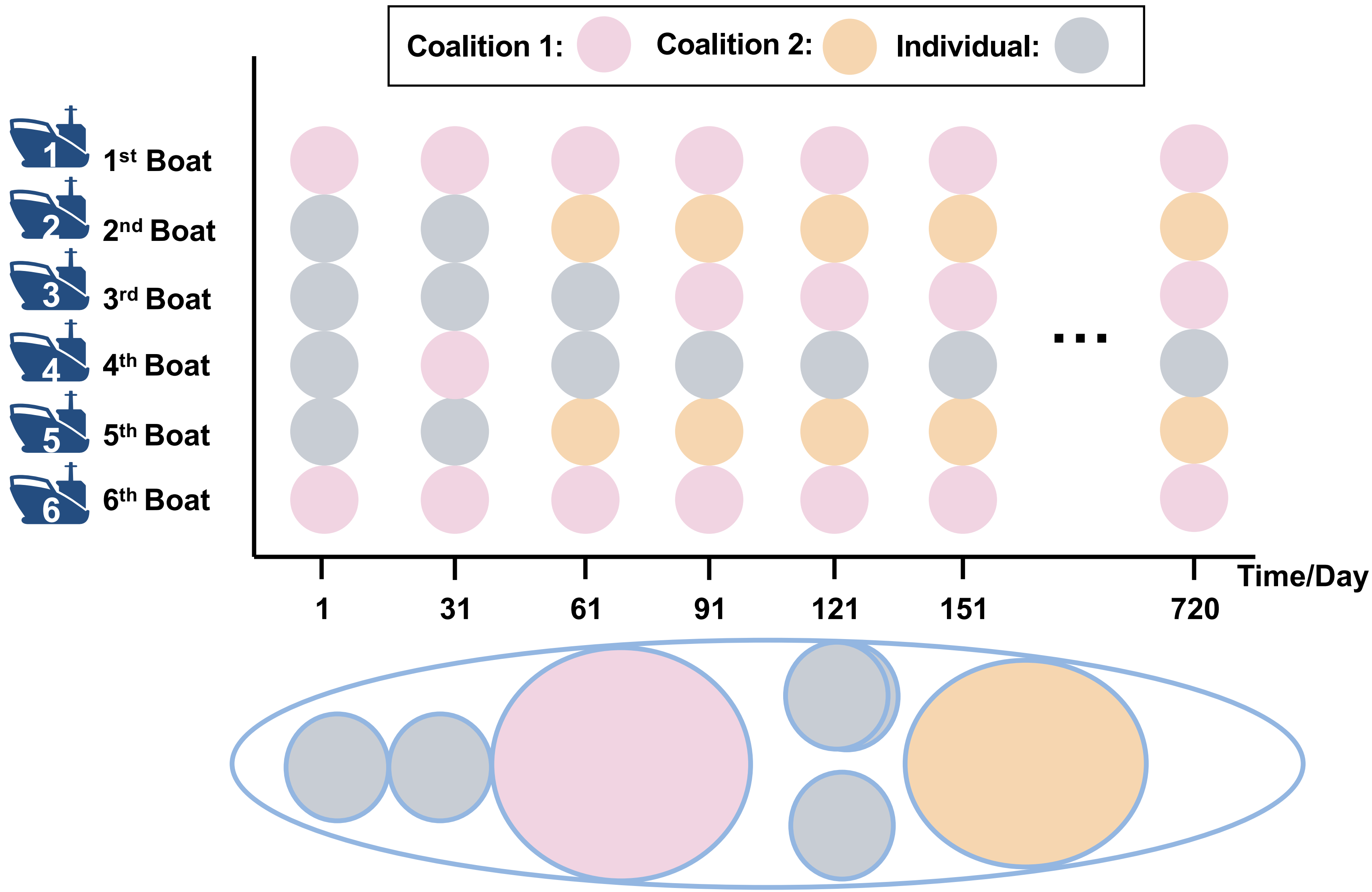
Grand Coalition

(Reasonable)

Description	Fish Value
total fish caught	343.25×10^3
1st boat	29.53×10^3
2nd boat	36.91×10^3
3rd boat	44.29×10^3
4th boat	55.36×10^3
5th boat	72.82×10^3
6th boat	103.34×10^3

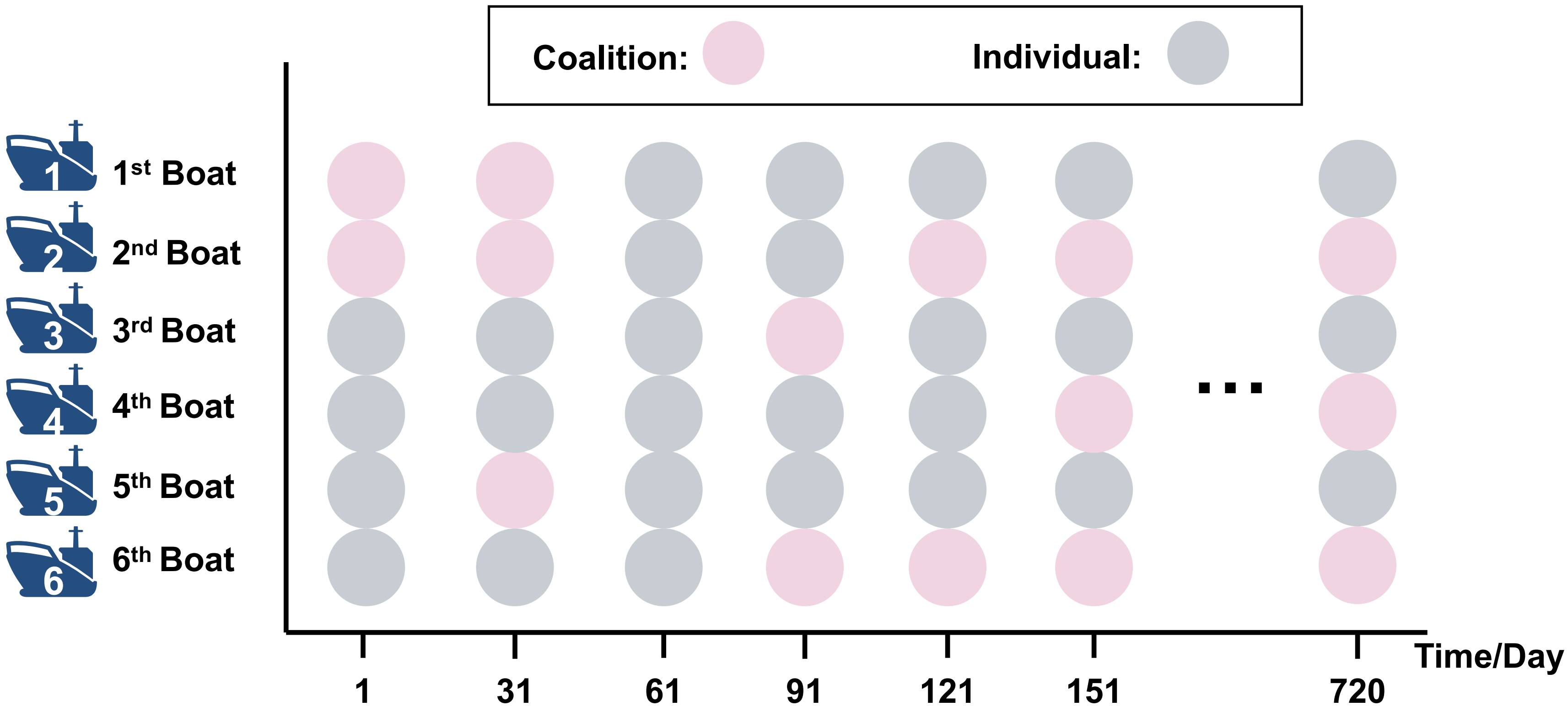
09 RESULT: CONTROLLED COALITION

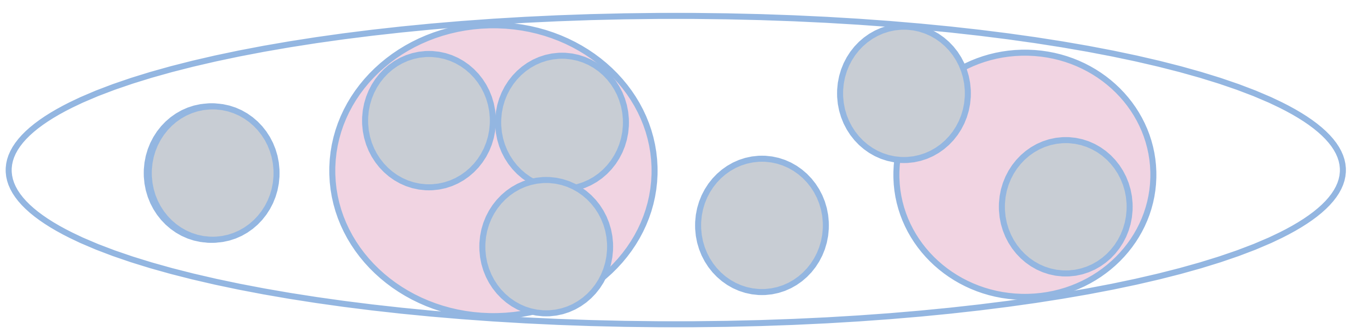
- Redistribution (A Compensation Mechanism)



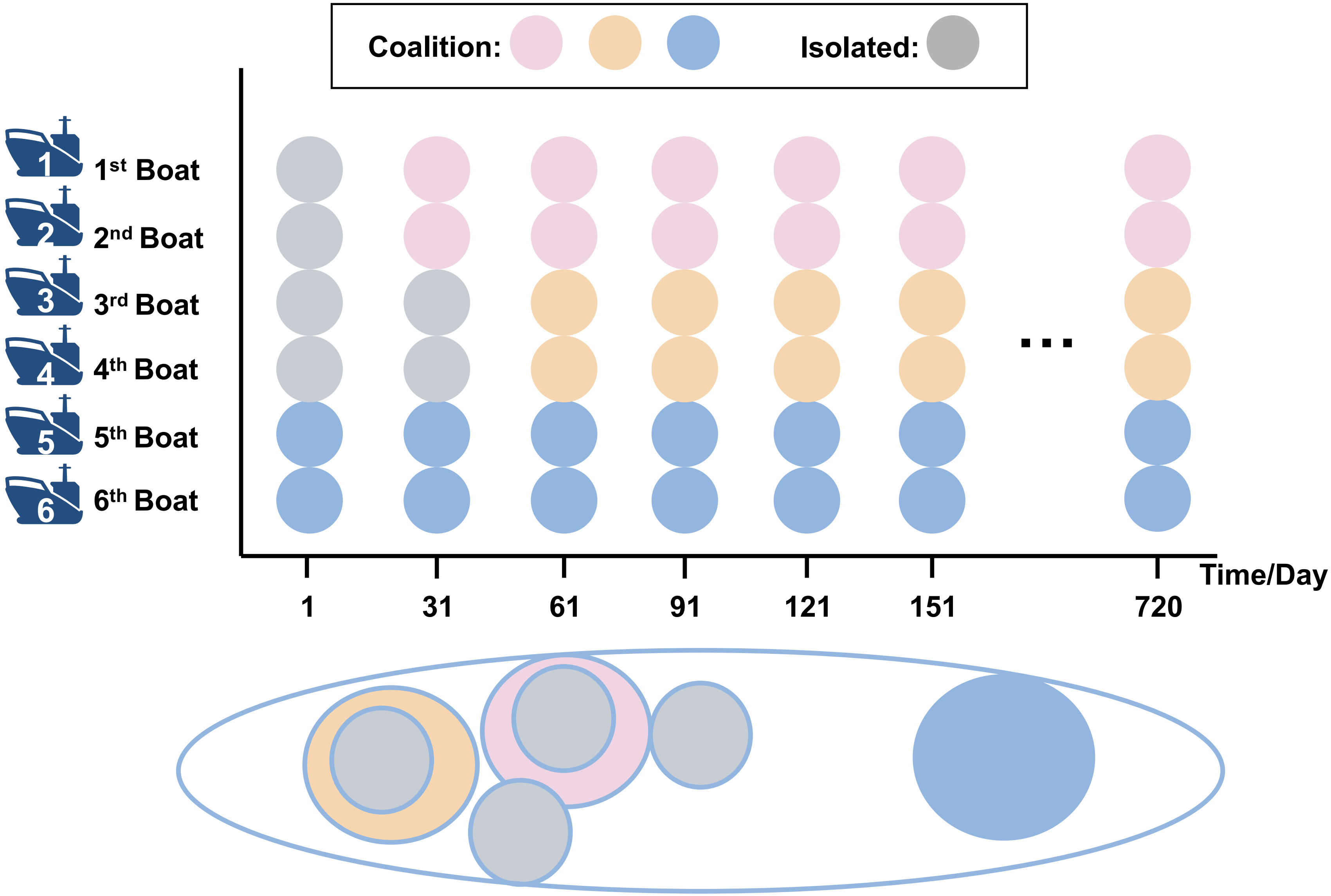
10 RESULT: CONTROLLED COALITION

- Without Redistribution





11 RESULT: HEURISTIC CONTROLLED



12 RESULT: HEURISTIC CONTROL

- Heuristic

Symbol	Description	Fish Value
$\mathcal{F}$	total fish caught	342.45×10^3
$\mathcal{F}_1$	1st boat	29.46×10^3
$\mathcal{F}_2$	2nd boat	36.82×10^3
$\mathcal{F}_3$	3rd boat	44.19×10^3
$\mathcal{F}_4$	4th boat	55.23×10^3
$\mathcal{F}_5$	5th boat	73.65×10^3
$\mathcal{F}_6$	6th boat	103.10×10^3

Heuristic Coalition Control Model: The total fish caught of entire fleet and each boat in 2 years/720 time steps

- Comparing with other methods

Method	Fish Value
Grand	343.25×10^3
Isolated	337.22×10^3
Controlled	343.00×10^3
Heuristic	342.45×10^3

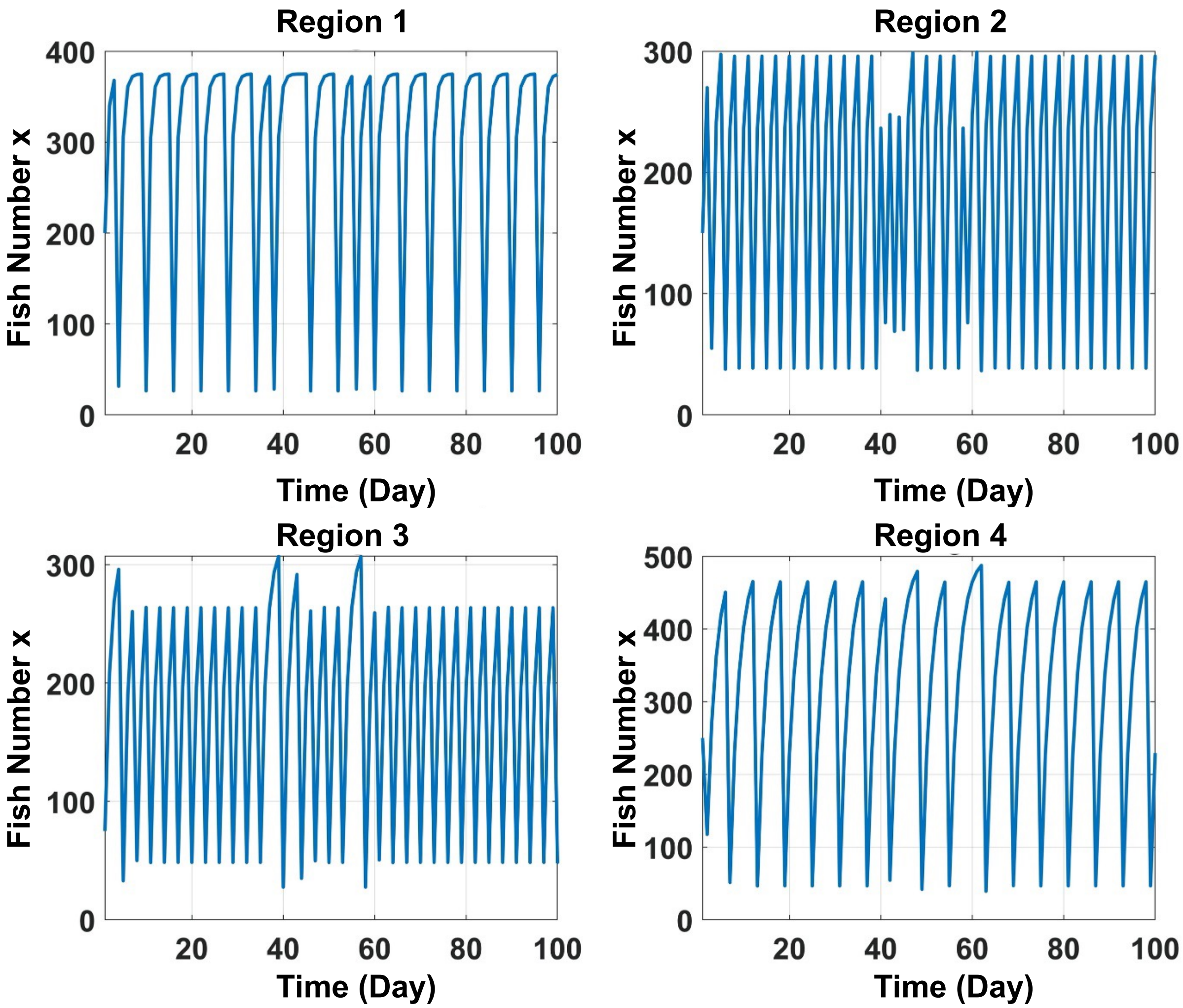
Summary of total fish caught Compare: each coalition control method in 2 years/ 720 time steps.

Method	4×6	4×12	4×18	4×24
Grand	6.1s	25.8s	156.1s	197.3s
Isolated	3.0s	6.6s	9.6s	19.1s
Controlled	382.7s	6361.9s	NA	NA
Heuristic	14.5s	46.9s	71.2s	115.2s

Running time Compare: different coalitions for one iteration

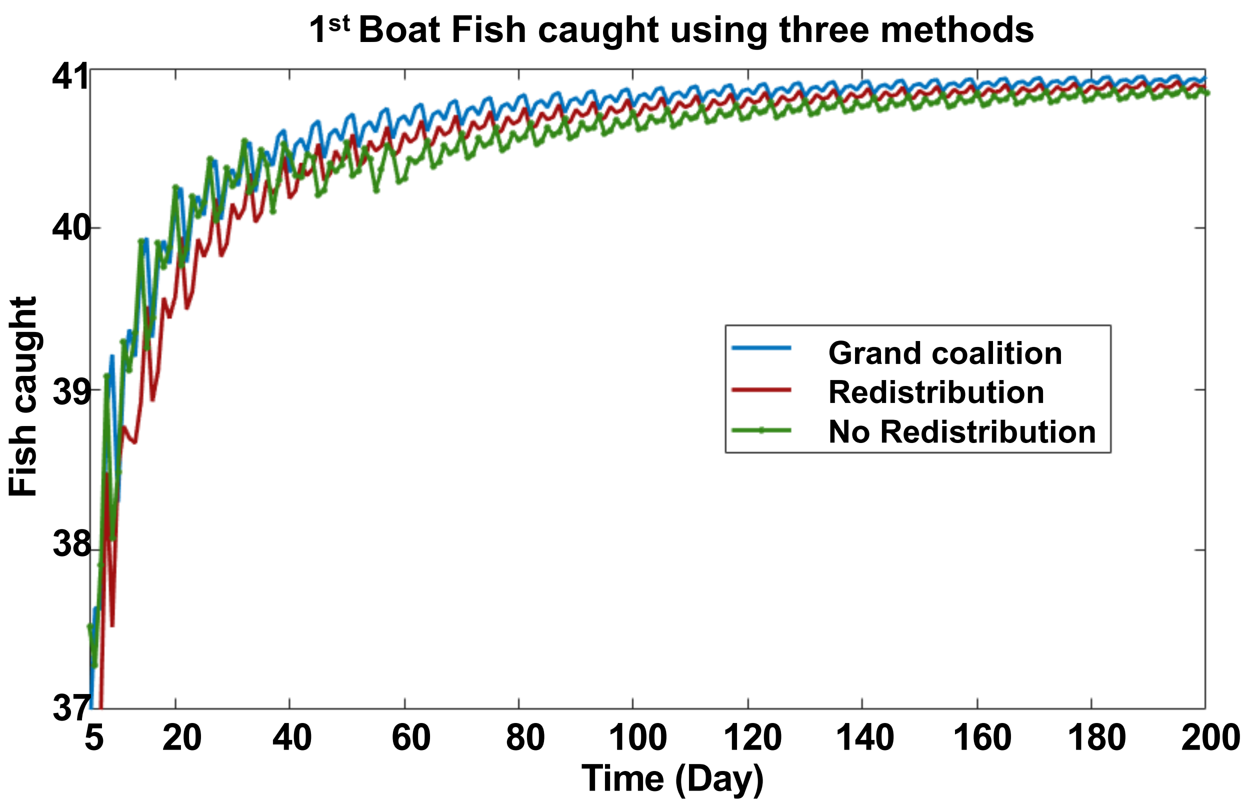
13 RESULT: SUSTAINABILITY

Changes in fish numbers in each region without redistribution



- Fish **fill in quickly** once the fish in a region is depleted.
- The **periodic behavior** changes along with the **change of coalition structure**.
- A **dynamic equilibrium state** is maintained.

14 RESULT: DIFFERENCES



- The periodic solutions of three cases all converge to **almost the same point**.
- **Without global information**, one agent can still plan through **communication** and cooperation with other agents in our controlled coalition approaches.

15 PSEUDO-CODE

Listing 1: Ps
solutions usin

Inputs: fish p
temporary
iteration

if $c = 1$ % gra
max $= 1$
end if

$\mathbf{x}^{eq} = \mathbf{x}$

for it $= 1$ to m
for $i = 1$ t

$\mathcal{O}^*_{\mathcal{K}} \leftarrow$

$\tilde{\mathbf{E}}_{\mathcal{C}_i}$ an
end for
end for

$\mathbf{E}^{eq} = \begin{bmatrix} \tilde{\mathbf{e}}_{\mathcal{C}_1} \\ \tilde{\mathbf{e}}_{\mathcal{C}_2} \\ \vdots \\ \tilde{\mathbf{e}}_{\mathcal{C}_c} \end{bmatrix}$ … $\lfloor \mathbf{e}^{eq}_{\mathcal{C}_c} \rfloor$

Output: vector $\mathbf{E}^{eq}$, $\mathbf{x}^{eq}$

Listing 2: Pseudo-code

Inputs: coalition structu
condition of merging

if $c > 1$
$\mathcal{C}^{temp}_{\mathcal{K}} = \mathcal{C}_{\mathcal{K}}$
for $i = 1$ to $c - 1$ d
for $j = i + 1$ to
$\mathcal{C}^{new}_{\mathcal{K}} \leftarrow$ me

Call algorithm
$\mathcal{C}^{temp}_{\mathcal{K}}$

if $\mathscr{M}$ meets
$\mathcal{C}^{temp}_{\mathcal{K}} =$
end if
end for
end for
$\mathcal{C}^*_{\mathcal{K}} = \mathcal{C}^{temp}_{\mathcal{K}}$
else
$\mathcal{C}^*_{\mathcal{K}} = \mathcal{C}_{\mathcal{K}}$
end if

Output: matrix $\mathcal{C}^*_{\mathcal{K}}$

Listing 3: Pseudo-code illustration of spl

Input: coalition structure $\mathcal{C}_{\mathcal{K}}$, number of coa
condition of splitting $\mathscr{S}$

$\mathcal{C}^{temp}_{\mathcal{K}} = \mathcal{C}_{\mathcal{K}}$
for $i = 1$ to $c - 1$ do
if $c(\mathcal{C}_i) > 2$
for $j = 1$ to $c(\mathcal{C}_i)$
$\mathcal{C}^{new}_{\mathcal{K}} \leftarrow$ split $\mathcal{C}_j$ in $\mathcal{C}^{temp}_{\mathcal{K}}$

Call algorithm from Listing 1 for
$\mathcal{C}^{temp}_{\mathcal{K}}$

if $\mathscr{S}$ meets
$\mathcal{C}^{temp}_{\mathcal{K}} = \mathcal{C}^{new}_{\mathcal{K}}$
end if
end for
end if
end for
$\mathcal{C}^*_{\mathcal{K}} = \mathcal{C}^{temp}_{\mathcal{K}}$

Output: matrix $\mathcal{C}^*_{\mathcal{K}}$

Listing 4: Pseudo-code illustration of hierarchical clustering method

Input: individual boat efforts $\mathbf{E}$, coalition structure $\mathbf{V}$,
number of coalitions c, maximum iteration max,
coalitions thresholds ϵ, trade-off parameters μ, γ

for it $= 1$ to max do
$\mathbf{V} = \mathbf{E}$
for $i = 1$ to $c - 1$ do
solve $\mathbf{v}_i = \frac{1}{\gamma |V_k| - \mu} (\gamma \sum_{l \in V_k} \mathbf{v}_l - \mu \mathbf{e}_k)$
end for
$\mathbf{V}^{temp} = \mathbf{V}$
for $i = 1$ to $c - 1$ do
for $j = i + 1$ to c do
dist $= \|\mathbf{v}_i - \mathbf{v}_j\|_2^2$
if dist $< \epsilon$
$\mathbf{V}^{new} \leftarrow$ merging $\mathcal{C}_i$ and $\mathcal{C}_j$ in $\mathbf{V}^{temp}$
end if
end for
end for

Call algorithm from Listing 1 for $\mathbf{E}^{new}$
(modify the objective function to
$-O_{\mathbf{E}} + \mu \sum_i \|\mathbf{e}_i - \mathbf{v}_i\|_2^2$)

if $O_{\mathbf{E}}$ is not increasing or satisfying specified conditions
$\mathbf{V} = \mathbf{V}^{new}$
$\mathbf{E} = \mathbf{E}^{new}$
break
end if
end for

Output: matrix $\mathbf{E}$, matrix $\mathbf{V}$

16 CONCLUSION

- Both globally optimal and heuristic approaches can automatically **adjust the coalition structure** to get optimized results.
- Coalition control method results are **close to** that of the grand coalition that finds a social optimum solution, whereas our algorithm reaches the equilibrium through the **Nash-bargaining process**.
- Our methods can be applied to the scenarios in which **individual interest and collective interest conflict**. It will solve the problem **both fairly and reasonably** based on **communication**.

01 Fish Models

1. **Nonlinear fish evolution equation**
2. **Species value classification**

…

02 Boats

1. **Competition mechanism**
2. **Fishing costs…**

03 Algorithm

1. **Additional coalition conditions**
2. **Computational optimization of MPC…**

04 Applications

Transportation, irrigation, power grids…

Thanks!
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Weizhi Du

Coalition Control Model: A Dynamic Resource Distribution Method Based on Model Predictive Control

arXiv:2011.12711 [cs.MA]

GitHub: https://github.com/weizhi-du/coalition-control-model